Teaching briefs... Shortest Connection Problems: Applied Computation

by Susan H. Picker

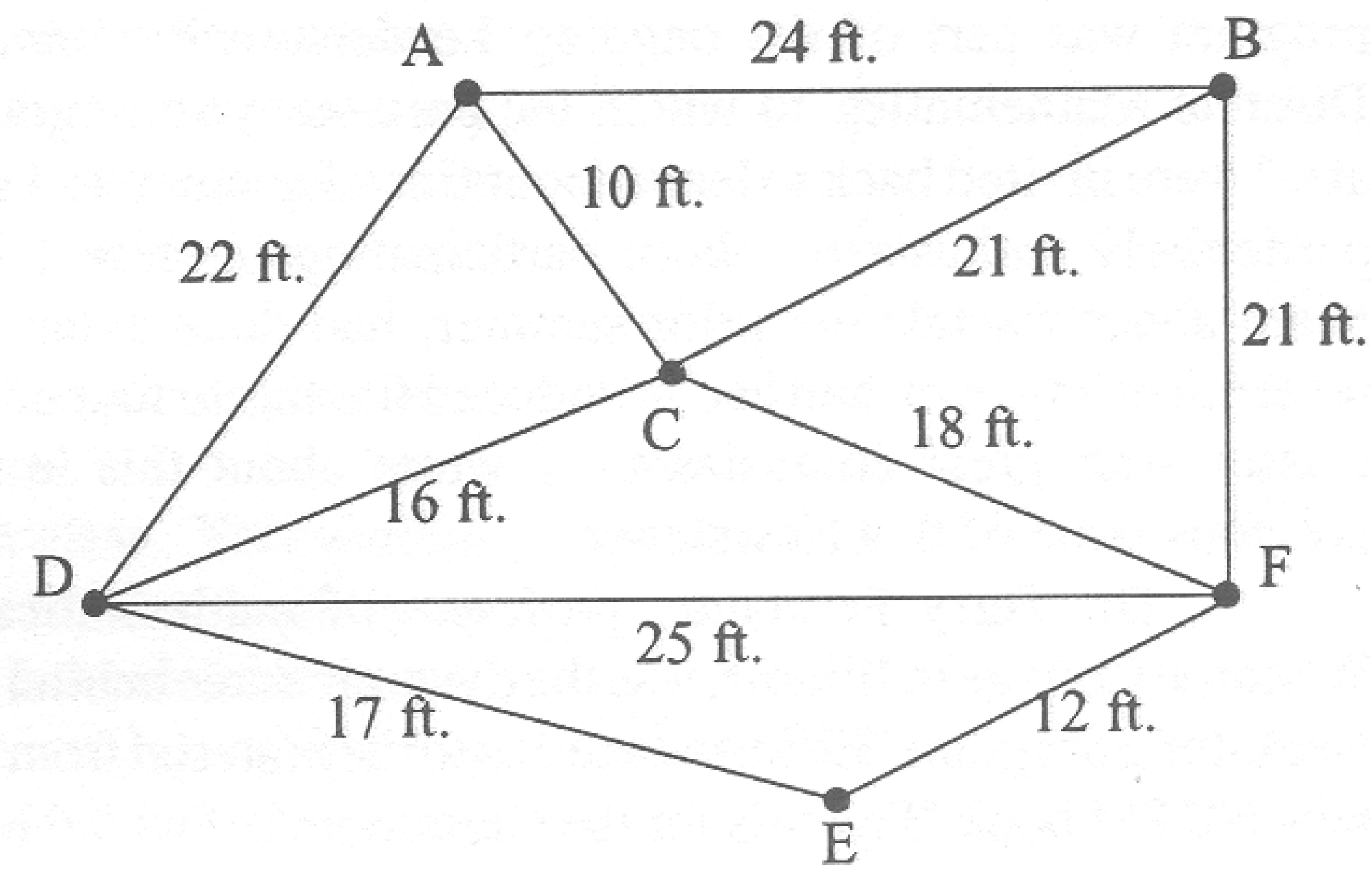
For students who are weak in computational skills, shortest connection problems which use weighted graphs are an excellent way to reinforce arithmetic concepts and introduce applied problem solving. Too often, students with weak skills are made to do pages of a particular problem or are placed in front of computers which generate one example after another and praise correct answers. It is no wonder that many students come to think of mathematics as a stifling subject.

Shortest connection problems provide interesting real-life problems. To solve them, computation is necessary, but only as the means for arriving at a solution. Among the other advantages of these problems: there often is more than one way to arrive at the solution; the problems require students to work with and understand an abstract model - a graph; they are excellent problems to use with cooperative learning and writing in the classroom; and they give rise to wonderful discussions about efficiency, as students seek a minimum length of wire or pipeline or cable.

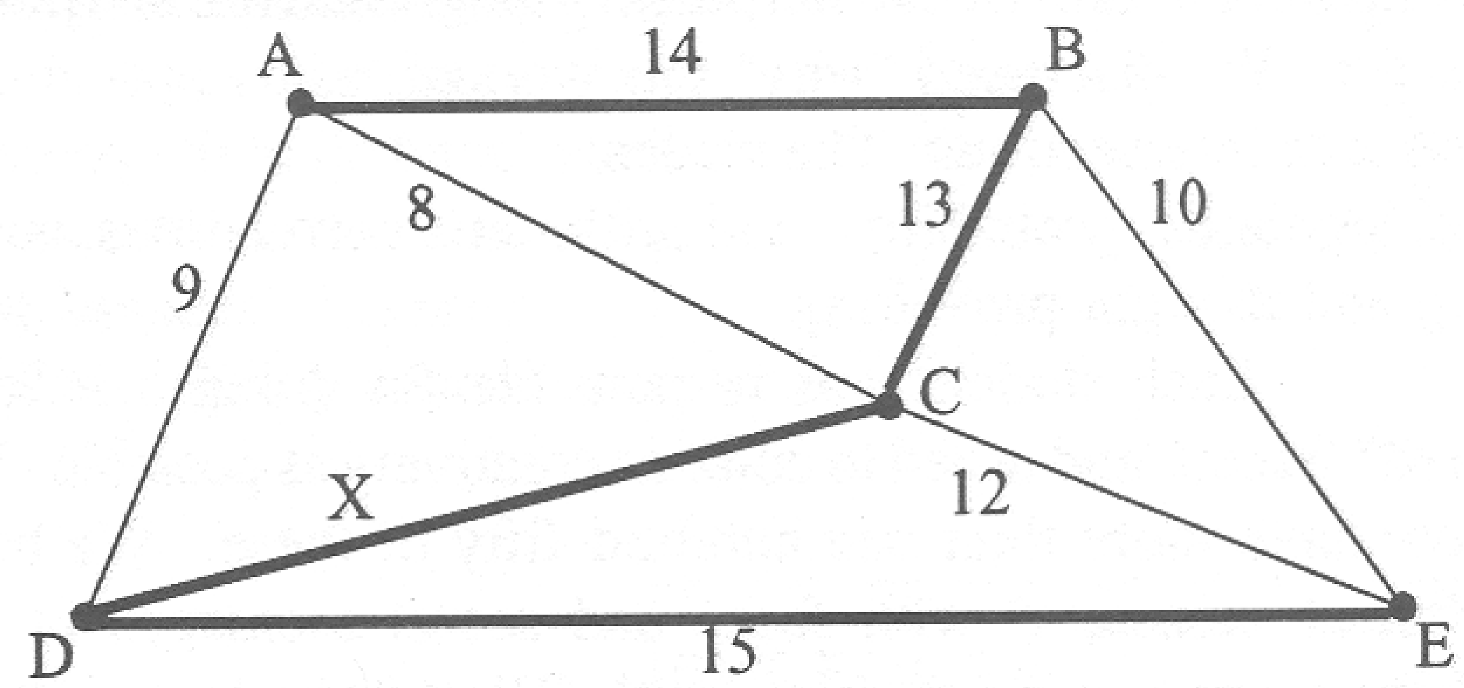
I give students a few of the problems and encourage them to try to find a rule for finding the shortest connection in the graph. Working together, students often come to see that the most efficient technique is always to link the vertex which is nearest to any vertex already connected in the solution.

In the two problems below, which are from The Decision Maths Pack, (The Spode Group, Australian Edition by P. Galbraith & A. Carr, Edward Arnold, Australia, 1988) extra information must be generated by the student. In #1, the student has to convert from yards to feet to reach a conclusion as to whether the available cable is sufficient. In #2, students have to use algebra to determine the amount of wire at x, then see if there is a shorter connection possible. Of course if students are not familiar with algebra, the unknown length can be given explicitly. (See "*Solutions...*" on page 6.) ■

1. Jack and Jill plan to hang electric lanterns in their backyard for a party. They have 25 *yards* of cable. Will this be enough to stretch between the six points shown?



2. Andrew, Bob, Crystal, Dwayne and Ezra are camping out near Bear Mountain in the pouring rain. They have all set up their tents and want to have an intercom system between them so that they can talk without getting wet. They have five transmitters and need to decide how to wire them. A possible wiring system is shown by the **dark** lines using 49 meters of wire. Can you find a way of connecting all five friends that uses *less* wire?



Topics... Two Problems Involving Graphs (reprise)

by Joseph G. Rosenstein

Last Newsletter's article with the above title posed two problems involving graphs. The first problem was solved but the second problem was left to the reader -- "A mouse eats her way through a 3x3x3 cube of cheese by tunnelling through all of the 27 1x1x1 minicubes. If she starts at one corner of the cube and always moves to an adjacent uneaten minicube, can she finish at the center of the cube?" The two problems were provided a common context -- that of bipartite graphs -- and readers were invited to see how the second problem fit into this context. The vertices of a bipartite graph can be colored using two colors -- say, red and black -- so that adjacent vertices have different colors. The 27 minicubes of cheese can be viewed as the 27 vertices of a graph, where two vertices are adjacent (in the graph theory sense) if the corresponding minicubes are adjacent (in the physical sense). This graph is bipartite -- see the diagram on page 1. Now the mouse wants to take a walk through this graph starting from a corner cube, ending at the center cube, and passing through each vertex exactly once. As she walks, she must always move from a red vertex to a black one, and vice versa. Thus if the first vertex on the walk is red, then the 27th vertex on the walk will also be red. This means that if the corner cube is red and the mouse can complete its task then the center cube must also be red. But in fact, it is black. Sorry, mouse, it can't be done! But can 25 children whose seats form a 5x5 square simultaneously move to adjacent seats? ■